QUESTIONS AND CONCLUSIONS

QUESTIONS 11-12

Q1:

A family consisting of a mother, father, and three children is attending a theater. They need to sit on 5 adjacent seats, and only the youngest child must sit between the mother and father.

How many different ways can they sit? **(1 point)**

A) 24 B) 20 C) 12 D) 6 E) 4

CONCLUSIONS

Q1: Since the youngest child must sit between the mother and father, we need to first focus on the three of them occupying three consecutive seats. There are two possible ways to arrange them:

1. Mother – Youngest Child – Father
2. Father – Youngest Child – Mother

Thus, there are 2 ways to arrange the mother, father, and youngest child in those 3 adjacent seats.

Once the mother, father, and youngest child are seated, there are 2 remaining seats for the other two children. These two children can be seated in $2! = 2$ ways.

We have 5 total seats and the group of three (mother, father, and youngest child) can sit in 3 different sets of adjacent seats. These 3 seats can start at positions 1, 2, or 3.

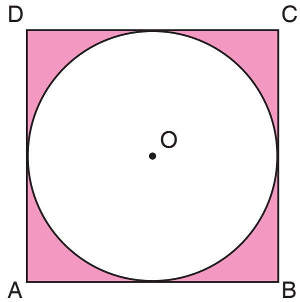
So, the correct total number of seating arrangements is:

Total ways $= 3 \times 2 \times 2! = 3 \times 2 \times 2 = 12$

The correct answer is C.

SOLUTION IS C

Q2:



In the figure, ABCD is a square, and the circle with center O is tangent to the sides of the square. It is given that $|AB| = 12$ cm.

What is the shaded area of the region between the square and the circle? **(2 points)**

A) $144 - 18\pi$ B) $144 - 36\pi$

C) $72 + 18\pi$ D) $72 - 18\pi$

E) $72 - 36\pi$

CONCLUSIONS

Q2: We are given that AB = 12 cm. Since ABCD is a square, all sides are equal, so the side length of the square is 12 cm.

The area A_{square} of the square is:

$$A_{square} = s^2 = 12^2 = 144 \text{ cm}^2$$

The circle is inscribed inside the square, meaning it touches all four sides of the square. The diameter of the circle is equal to the side length of the square.

The diameter of the circle is 12 cm,

Therefore, the radius r of the circle is: $r = \frac{12}{2} = 6$ cm

The area A_{circle} of the circle is given by the formula:

$$A_{circle} = \pi r^2 = \pi(6)^2 = 36\pi \text{ cm}^2$$

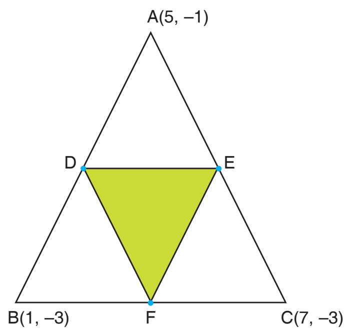
The area of the shaded region is the area of the square minus the area of the circle:

$$A_{shaded} = A_{square} - A_{circle} = 144 - 36\pi \text{ cm}^2$$

The area of the region between the square and the circle is $144 - 36\pi$ cm^2, so the correct answer is B.

SOLUTION IS B

Q3:



In the triangle ABC shown above, the points D, E, and F are the midpoints of the sides of triangle ABC.

What is the area of triangle DEF? (3 points)

A) $\frac{3}{2}$ B) 2 C) $\frac{5}{2}$ D) 3 E) $\frac{7}{2}$

CONCLUSIONS

Q3: We can use the formula for the area of a triangle when the vertices are given as coordinates:

Area of triangle

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substitute the coordinates of A(5, −1), B(1, −3), and C(7,−3):

Area of triangle

$$ABC = \frac{1}{2}\ 5(-3 + 3) + 1(-3 + 1) + 7(-1 + 3)$$

Simplifying:

$$= \frac{1}{2}|5(0) + 1(-2) + 7(2)| = 12|0 - 2 + 14|$$

$$= \frac{1}{2} \times 12 = 6 \text{ square units.}$$

Since D, E, and F are midpoints of the sides of triangle ABC, triangle DEF is the medial triangle of triangle ABC. The area of the medial triangle is always one-fourth of the area of the original triangle.

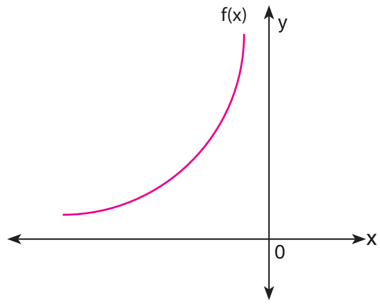
Thus, the area of triangle DEF is:

Area of triangle DEF $= \frac{1}{4} \times$ Area of triangle ABC $= \frac{1}{4} \times 6 = \frac{3}{2}$

The area of triangle DEF is 3/2 square units, so the correct answer is A.

SOLUTION IS A

Q4:



In the figure above, the graph of the function y = f(x) is given.

Based on the graph:

- I. f(x) is increasing.
- II. f(x) is negative.
- III. f(0)=0.

Which of the following statements are true?
(4 points)

A) Only I
B) Only II
C) I and II
D) I and III
E) II and III

CONCLUSIONS

Q4: The graph of y=f(x) is provided, and we need to determine which statements about the function are true based on the graph.

Analyze the statements:

Statement I: f(x) is increasing

From the graph, as x increases, f(x) clearly increases as well. The curve moves upwards as we go from left to right, indicating that the function is indeed increasing.

This statement is true.

Statement II: f(x) is negative

The graph shows that f(x) is entirely above the x-axis for all values of x shown. A function is considered positive when it's above the x-axis. Therefore, the function is positive, not negative.

This statement is false.

Statement III: f(0) = 0

From the graph, it does not appear that f(x) passes through the origin. It seems that f(0) has a positive value, not 0.

This statement is false.

Statement I is true, but both Statements II and III are false.

The correct answer is A.

SOLUTION IS A

Q5: $\frac{\cos^2 x + \sin^2 x + 1}{1 - \sin^2 x}$

Which of the following is the simplest form of the expression? (5 points)

A) 2
B) 2cotx
C) $\tan^2 x$
D) –2
E) –2tanx

CONCLUSIONS

Q5: We know the identity:

$1 - \sin^2 x = \cos^2 x$

So, we can substitute $\cos^2 x$ for $1 - \sin^2 x$ in the denominator of the expression.

Thus, the expression becomes:

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$

The numerator contains $\cos^2 x - \sin^2 x + 1$. We can rewrite the expressioncos²x – sin² x as cos(2x), using the double angle identity for cosine:

$\cos^2 x - \sin^2 x = \cos(2x)$.

Thus, the numerator becomes $\cos(2x) + 1$.

Now the expression becomes:

$$\frac{\cos(2x) + 1}{\cos^2 x}$$

We need to break this into two terms:

$$\frac{\cos(2x)}{\cos^2 x} + \frac{1}{\cos^2 x}$$

The second term is simply:

$$\frac{1}{\cos^2 x} = \sec^2 x$$

For the first term, recall that:

$\cos(2x) = 2\cos^2 x - 1$

So,

$$\frac{\cos(2x) + 1}{\cos^2 x} = 2 - \frac{1}{\cos^2 x}$$

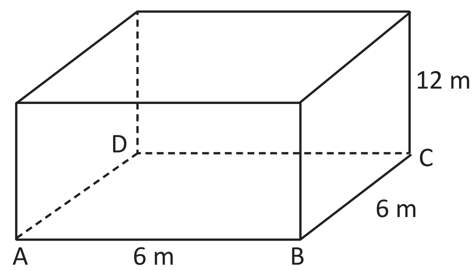
Therefore, the entire expression simplifies to:

$2 - \sec^2 x + \sec^2 x = 2$

The simplified expression is 2, so the correct answer is A..

SOLUTION IS A

Q6:



In the figure, a square pyramid is placed inside a rectangular prism, using the ABCD face as the base of the pyramid.

Given this information, what is the volume of the square pyramid? (6 points)

A) 100 cm^3
B) 110 cm^3
C) 115 cm^3
D) 120 cm^3
E) 144 cm^3

CONCLUSIONS

Q6: The formula for the volume V of a pyramid is:

$$V = \frac{1}{3} \times \text{Base Area} \times \text{Height}$$

Since the base of the pyramid is a square with side lengths of 6 meters, the area of the base is:

Base Area = 6 × 6 = 36 square meters

The height of the pyramid is the same as the height of the rectangular prism, which is 12 meters.

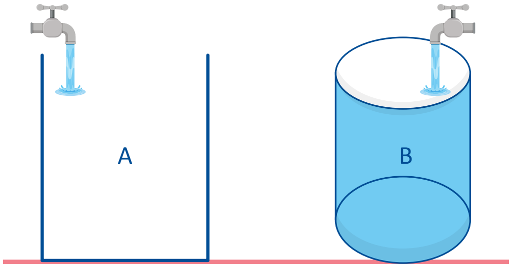
Now, using the formula for the volume of a pyramid:

$$V = \frac{1}{3} \times 36 \times 12 = \frac{1}{3} \times 432 = 144 \text{ cubic meters}$$

The volume of the square pyramid is 144 cubic meters, so the correct answer is E.

SOLUTION IS E

Q7:



The volume of tank A is $x^3 - x^2 - 6x$ liters, and the volume of tank B is $3x^2 + 9x$ liters.

A faucet that fills tank A releases water at a rate of x+2 liters per minute and fills the tank completely in m minutes. A different faucet that fills tank B releases water at a rate of x + 3 liters per minute and fills the tank completely in n minutes.

What is the expression for m + n? (7 points)

A) $x^2 - 3x$
B) $x^2 - 1$
C) $x^2 - x$
D) $x^2 - 3$
E) x^2

CONCLUSIONS

Q7: The volume of tank A is $x^3 - x^2 - 6x$ liters, and the faucet releases x + 2 liters per minute. The time m to fill tank A is given by:

$$m = \frac{\text{Volume of tank A}}{\text{Flow rate for tank A}} = \frac{x^3 - x^2 - 6x}{x + 2}$$

We can simplify this by performing polynomial division.

Polynomial Division: $\frac{x^3 - x^2 - 6x}{x + 2}$

Step 1: Divide the first term of the numerator by the first term of the denominator: $\frac{x^3}{x^2}$

Now multiply $x^2$by x + 2:

$x^2(x + 2) = x^3 + 2x^2$

Subtract $x^3 + 2x^2$ from $x^3 - x^2 - 6x$:

$(x^3 - x^2 - 6x) - (x^3 + 2x^2) = -3x^2 - 6x$

Step 2: Divide the first term of the new numerator by the first term of the denominator:

$\frac{-3x^2}{x} = -3x$

Now multiply −3x by x + 2: $-3x(x + 2) = -3x^2 - 6x$

Subtract $-3x^2 - 6x$ from $-3x^2 - 6x$:

$(-3x^2 - 6x) - (-3x^2 - 6x) = 0$

Thus, the division is exact, and: $m = {}^2 - 3x$

The volume of tank B is $3x^2 + 9x$ liters, and the faucet releases x + 3 liters per minute. The time n to fill tank B is:

$$n = \frac{\text{Volume of tank B}}{\text{Flow rate for tank B}} = \frac{3x^2 + 9x}{x + 3}$$

We can simplify this by factoring the numerator:

$$n = \frac{3x(x + 3)}{x + 3}$$

Cancel the common factor x + 3: n = 3x

Now, sum the values of m and n:

$m + n = (x^2 - 3x) + 3x = x^2$

The expression for m + n is x^2, so the correct answer is E.

SOLUTION IS E