## QUESTIONS 9-10

**Q1: What is the ratio of the product of eight 2's to the sum of sixty-four 2's? (1 point)**

A) 4 B) 1 C) 2 D) $\frac{1}{4}$ E) $\frac{1}{2}$

### CONCLUSIONS

**Q1:** We need to find the ratio of:

The product of eight 2's

The sum of sixty–four 2's

The product of eight 2's means multiplying 2 by itself eight times:

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 28 = 256$

The sum of sixty-four 2's means adding 2 together sixty-four times:

$2 + 2 + 2 + \ldots + 2 = 64 \times 2 = 128$

The ratio of the product of eight 2's to the sum of sixty-four 2's is:

$$\frac{2^8}{64} \times 2 = \frac{256}{128}$$

The correct answer is C.

*SOLUTION IS C*

**Q2:** The arithmetic mean of x, y, and z is 9. The arithmetic mean of x and y is 11.

**What is the value of z? (2 points)**

A) 5 B) 6 C) 7 D) 8 E) 9

### CONCLUSIONS

**Q2:** The arithmetic mean of x, y, and z is 9. This can be written as:

$$\frac{x + y + z}{3} = 9$$

4 pt Multiply both sides by 3:

$x + y + z = 27$

The arithmetic mean of x and y is 11.

This can be written as:

$$\frac{x + y}{2} = 11$$

Multiply both sides by 2:

$x + y = 22$

We have two equations:

$x + y + z = 27$

$x + y = 22$

Subtract equation 2 from equation 1:

$(x + y + z) - (x + y) = 27 - 22$,

which simplifies to: $z = 5$.

*SOLUTION IS A*

**Q3:**


A rectangle with an area of $(x^2 + 8x + 15)$ is divided into four rectangular regions as shown in the figure above.

**According to the polynomial P(x) representing the total area of the yellow regions, what is the polynomial P(x + 4)? (3 points)**

A) 8x + 32 B) 8x + 16
C) 4x + 16 D) 4x + 8
E) 3x + 6

**CONCLUSIONS**

**Q3:** The total area of the large rectangle is given as $(x^2 + 8x + 15)$. This rectangle is divided into four smaller rectangles, and we are specifically interested in the two yellow regions.

The dimensions of the large rectangle are:

- Height = x + 3,
- Width = x + 5, since:

  $(x + 3)(x + 5) = x^2 + 8x + 15$

There are two yellow regions in the figure:

- The yellow rectangle at the top-left has dimensions 3 × x, so its area is: 3x
- The yellow rectangle at the bottom-right has dimensions x × 5, so its area is: 5x

The total area of the yellow regions is the sum of the two areas:

Total yellow area = 3x + 5x = 8x. Thus, P(x) = 8x.

Now, substitute x + 4 into the polynomial

P(x) = 8x:

$P(x + 4) = 8(x + 4)$

First, expand the terms:

$8(x + 4) = 8x + 32$

The correct option is A.

*SOLUTION IS A*

**Q4:** $\dfrac{\sin60° + \cos45°}{\cos30° + \sin45°}$

**Evaluate the expression. (4 points)**

A) 1 B) 2 C) 3 D) 4 E) 5

**CONCLUSIONS**

**Q4:** $\sin60°: \dfrac{\sqrt{3}}{2}$

$\cos45°: \dfrac{\sqrt{2}}{2}$

$\cos30°: \dfrac{\sqrt{3}}{2}$

$\sin45°: \dfrac{\sqrt{2}}{2}$

Now, substitute the values of the trigonometric functions into the expression:

$$\frac{\sin60° + \cos45°}{\cos30° + \sin45°} = \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}}$$

The numerator and the denominator are identical, so the expression simplifies to:

$$\frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}} = 1$$

*SOLUTION IS A*

**Q5:**

$$\frac{2x+1}{x+3} = \frac{x+4}{3x+1}$$

**What is the product of the roots of the equation obtained from this expression? (5 points)**

A) $\frac{14}{5}$ B) $\frac{13}{5}$ C) $\frac{12}{5}$ D) $\frac{11}{5}$ E) 2

**CONCLUSIONS**

**Q5:** Cross-multiplying the terms to eliminate the denominators gives:

$(2x + 1)(3x + 1) = (x + 3)(x + 4)$

Now expand both sides:

- Left side:

  $(2x + 1)(3x + 1) = 2x(3x) + 2x(1) + 1(3x) + 1(1)$
  $= 6x^2 + 2x + 3x + 1 = 6x^2 + 5x + 1$

- Right side:

  $(x + 3)(x + 4) = x(x) + x(4) + 3(x) + 3(4)$
  $= x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

Now, we have:

$6x^2 + 5x + 1 = x^2 + 7x + 12$

Subtract $x^2 + 7x + 12$ from both sides:

$6x^2 + 5x + 1 - (x^2 + 7x + 12) = 0$

Simplifying:

$6x^2 - x^2 + 5x - 7x + 1 - 12 = 0$

$5x^2 - 2x - 11 = 0$

In a quadratic equation of the form $ax^2 + bx + c = 0$, the product of the roots is given by:

Product of roots $= \frac{c}{a}$

For the equation $5x^2 - 2x - 11 = 0$

- $a = 5$,
- $c = 11$

Thus, the product of the roots is: $\frac{11}{5}$

*SOLUTION IS D*

**Q6:** $\frac{24}{13} + \frac{23}{12} + \frac{22}{13} + \frac{21}{12} + \ldots + \frac{2}{13} + \frac{1}{12}$

**Find the result of the sum of the following rational numbers. (6 points)**

A) 12 B) 16 C) 19 D) 20 E) 24

**CONCLUSIONS**

**Q6:** We will split the terms into two groups:

Terms with a denominator of 13:

$\frac{24}{13} + \frac{22}{13} + \frac{20}{13} + \ldots + \frac{2}{13}$

Terms with a denominator of 12:

$\frac{23}{12} + \frac{21}{12} + \ldots + \frac{1}{12}$

The terms with a denominator of 13 form an arithmetic sequence:

$\frac{24}{13} + \frac{22}{13} + \frac{20}{13} + \ldots + \frac{2}{13}$

This sequence has 12 terms, with a common difference of $-2$, and the first term is $\frac{24}{13}$ and the last term is $\frac{2}{13}$.

The sum of these terms is:

$S = \frac{n}{2} \times$ (first term + last term), where $n = 12$,

the number of terms. Substituting the values:

$S = \frac{12}{2} \times \frac{24}{13} + \frac{2}{13} = 6 \times \frac{26}{13} = 6 \times 2 = 12$

The terms with a denominator of 12 form another arithmetic sequence:

$\frac{23}{12} + \frac{21}{12} + \ldots + \frac{1}{12}$

This sequence also has 12 terms, with a common difference of $-2$, and the first term is 23/12 and the last term is $\frac{1}{12}$.

The sum of these terms is:

$S = \frac{12}{2} \times \frac{23}{12} + \frac{1}{12} = 6 \times \frac{24}{12} = 6 \times 2 = 12$

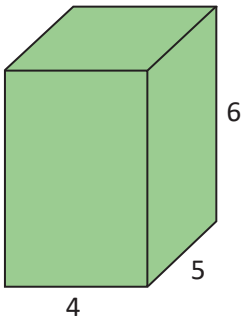
Now, adding the sums of the two groups:

$12 + 12 = 24$.

The correct answer is 24, so the correct option is E.

*SOLUTION IS E*

**Q7:**




A rectangular prism with dimensions of 4 units, 5 units, and 6 units is completely painted.

**If this wooden block is cut into 120 unit cubes, how many of these unit cubes will have no painted faces? (7 points)**

A) 12 B) 18 C) 24 D) 30 E) 36

## CONCLUSIONS

**Q7:** We are given a rectangular prism with the following dimensions:

- Length = 4 units
- Width = 5 units
- Height = 6 units

The entire surface of this prism is painted. After the block is cut into unit cubes, we need to find how many of these cubes will have no painted faces. These cubes are the ones completely inside the block, not touching the outer surface. The total number of unit cubes is simply the product of the dimensions of the rectangular prism:

Total number of cubes = $4 \times 5 \times 6 = 120$ unit cubes

The cubes that have no paint are the ones that are not touching the outer surface of the rectangular prism. These cubes are inside the block, and to count them, we must remove the outer layers of cubes from each face.

To do this, we subtract 1 unit from each dimension to remove the outer layer:

- The inner length becomes $4 - 2 = 2$ (since we are removing 1 unit from both sides),
- The inner width becomes $5 - 2 = 3$,
- The inner height becomes $6 - 2 = 4$.

Thus, the number of inner cubes with no painted faces is: Inner cubes = $2 \times 3 \times 4 = 24$

The number of unit cubes with no painted faces is 24, so the correct answer is C.

*SOLUTION IS C*