QUESTIONS

QUESTIONS 3-4

Q1: **What is the largest even natural number less than 500 whose digits are all different from each other?** **(1 point)**

A) 492 B) 498 C) 486 D) 482

Q2: In the four-digit number 3ΔΔ5, the sum of all digits is 20.

What is the sum of the digits Δ? **(2 points)**

A) 6 B) 8 C) 10 D) 12

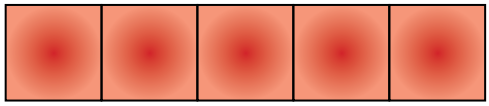
Q3:

A total of 845 people attended the concert in one day.

If 342 people attended in the morning and 276 people attended in the afternoon, how many people attended the concert in the evening? **(3 points)**

A) 207 B) 217 C) 227 D) 237

Q4:



The shape above consists of 5 equal squares.

If the perimeter of one square is 20 cm, what is the perimeter of the shape below in cm? **(4 points)**

A) 60 cm B) 84 cm
C) 72 cm D) 100 cm

Q5:

Alex went to bed at 21:30 in the evening and woke up at 07:15 in the morning.

How many hours did Alex sleep? (5 points)

A) 9 hours 15 minutes
B) 9 hours 45 minutes
C) 10 hours 15 minutes
D) 10 hours 45 minutes

Q6:

Emma wants to plant flowers around her rectangular garden. The long side of the garden is 60 meters, and the short side is 40 meters.

If a flower is planted at each corner and flowers are placed at equal intervals of 4 meters, how many flowers can be planted in total? (6 points)

A) 50 B) 70 C) 100 D) 110

Q7:

Chris, Taylor, Morgan, and Jamie participated in a 240-meter swimming race. The distances they covered in the same amount of time are as follows:

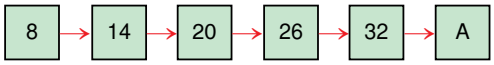
- Chris completed two-twelfths of the total distance.
- Taylor completed one-third of the total distance.
- Morgan completed one-fifth of the total distance.
- Jamie completed one-sixth of the total distance.

Who among them covered the same distance in the same amount of time? (7 points)

A) Chris – Taylor
B) Morgan – Jamie
C) Chris – Jamie
D) Taylor – Morgan

QUESTIONS 5-6

Q1:



Which of the following natural numbers should be written in place of A? (1 point)

A) 34 B) 36 C) 38 D) 40

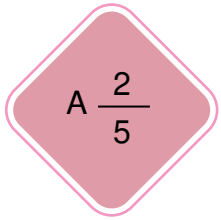
Q2:

$$\begin{array}{r} A\,B\,5 \\ -\;6\,B\,C \\ \hline 2\,C\,6 \end{array}$$

According to the given subtraction, what is the result of A + C? (2 points)

A) 15 B) 16 C) 17 D) 18

Q3:

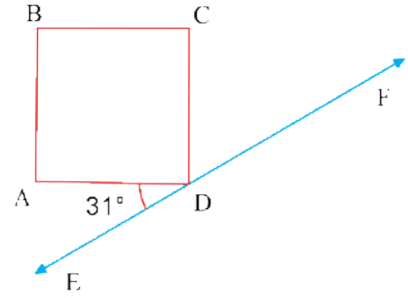


The mixed number above is converted into an improper fraction, resulting in $\frac{37}{5}$.

What is the value of A? (3 points)

A) 7 B) 8 C) 9 D) 10

Q4:



ABCD is a square, and the angle ($\widehat{ADE}$) measures 31°. Points E, D, and F are collinear (lie on the same line).

What is the measure of m($\widehat{CDF}$)? (4 points)

A) 31° B) 59° C) 69° D) 90°

Q5:

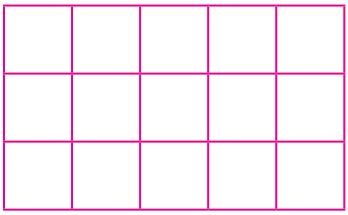
After Brian drives 10% of his journey, he has 45 km left to go.

Based on this information, how many kilometers is Brian's total journey? (5 points)

A) 50 km B) 55 km

C) 60 km D) 70 km

Q6:



In the figure composed of equal squares, the total length of the line segments of equal length is 190 cm.

What is the area of this rectangular shape? (6 points)

A) 375 cm^2 B) 350 cm^2

C) 325 cm^2 D) 300 cm^2

Q7:

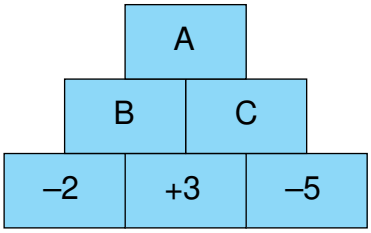
The price of 4 kilograms of apples is equivalent to the price of either 3 kilograms of bananas or 5 kilograms of tomatoes.

Based on this, what is the ratio of the price of 1 kilogram of tomatoes to the total price of 1 kilogram of bananas and 1 kilogram of apples? (7 points)

A) $\frac{20}{27}$ B) $\frac{12}{35}$ C) $\frac{15}{22}$ D) $\frac{12}{37}$

QUESTIONS 7-8

Q1:



In the figure above, each number in the top box is equal to the sum of the two numbers in the boxes directly below it.

What is the value of A + B + C? (1 point)

A) 2 B) 0 C) –2 D) –3

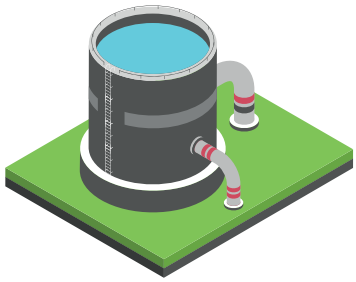
Q2:

I	II	III	IV
64^1	$(-8)^2$	$(-2)^6$	$(-4)^3$
1^{64}	4^3	8^2	$(-2)^6$

In the table above, which columns have equal results for the numbers in the top and bottom boxes? (2 points)

A) I and II B) I and III
C) II and III D) III and IV

Q3:

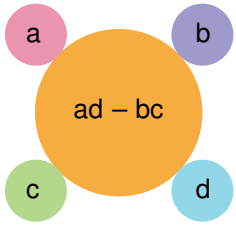


A tank is filled with water to two-sevenths of its capacity. After adding 45 liters of water, the tank becomes half full.

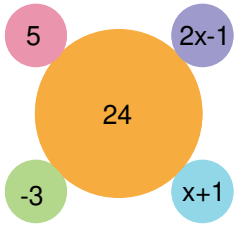
What is the total capacity of the tank? (3 points)

A) 105 B) 196 C) 210 D) 240

Q4: In the circular figure below, the number inside the large circle is written according to a specific rule.



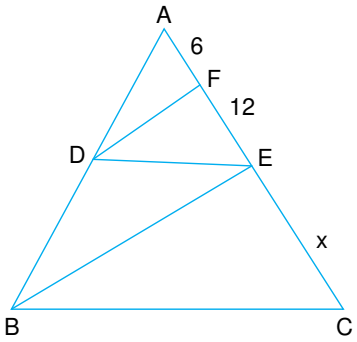
This rule is defined as follows:



According to this, what is the value of x? (4 points)

A) –1 B) 1 C) 2 D) 3

Q5:



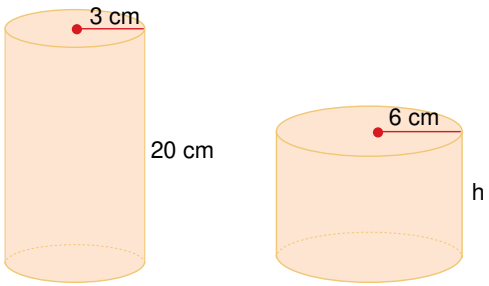
In triangle ABC, it is given that:

- DF // BE and DE // BC,
- IAFI = 6 cm,
- IFEI = 12 cm,
- IECI = x cm.

According to the information given, what is the value of x? (5 points)

A) 12 cm B) 24 cm
C) 36 cm D) 48 cm

Q6:

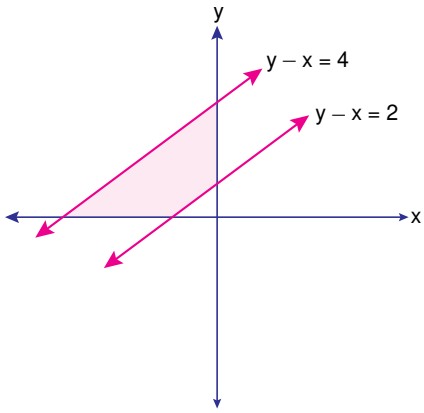


A cylinder with a base radius of 3 cm and a height of 20 cm is completely filled with water. This water is then poured into another cylinder with a base radius of 6 cm, filling it completely.

What is the height h of the second cylinder? (6 points)

A) 4 cm B) 5 cm
C) 10 cm D) 20 cm

Q7:



In the coordinate system above, the graphs of the lines y – x = 2 and y – x = 4

Based on this information, what is the area of the shaded region? (7 points)

A) 2 B) 4 C) 6 D) 8

QUESTIONS 9-10

Q1: **What is the ratio of the product of eight 2's to the sum of sixty-four 2's? (1 point)**

A) 4 B) 1 C) 2 D) $\frac{1}{4}$ E) $\frac{1}{2}$

Q2: The arithmetic mean of x, y, and z is 9. The arithmetic mean of x and y is 11.

What is the value of z? (2 points)

A) 5 B) 6 C) 7 D) 8 E) 9

Q3:

A rectangle with an area of $(x^2 + 8x + 15)$ is divided into four rectangular regions as shown in the figure above.

According to the polynomial P(x) representing the total area of the yellow regions, what is the polynomial P(x + 4)? (3 points)

A) 8x + 32 B) 8x + 16
C) 4x + 16 D) 4x + 8
E) 3x + 6

Q4: $\frac{\sin 60° + \cos 45°}{\cos 30° + \sin 45°}$

Evaluate the expression. (4 points)

A) 1 B) 2 C) 3 D) 4 E) 5

Q5:

$$\frac{2x+1}{x+3} = \frac{x+4}{3x+1}$$

What is the product of the roots of the equation obtained from this expression? (5 points)

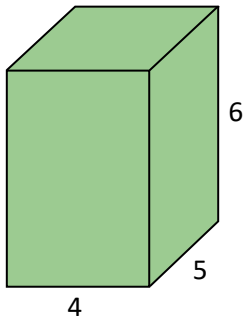
A) $\frac{14}{5}$ B) $\frac{13}{5}$ C) $\frac{12}{5}$ D) $\frac{11}{5}$ E) 2

Q6: $\frac{24}{13} + \frac{23}{12} + \frac{22}{13} + \frac{21}{12} + \ldots + \frac{2}{13} + \frac{1}{12}$

Find the result of the sum of the following rational numbers. (6 points)

A) 12 B) 16 C) 19 D) 20 E) 24

Q7:



A rectangular prism with dimensions of 4 units, 5 units, and 6 units is completely painted.

If this wooden block is cut into 120 unit cubes, how many of these unit cubes will have no painted faces? (7 points)

A) 12 B) 18 C) 24 D) 30 E) 36

QUESTIONS 11-12

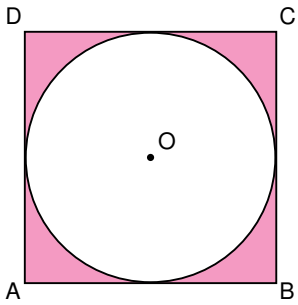
Q1:

A family consisting of a mother, father, and three children is attending a theater. They need to sit on 5 adjacent seats, and only the youngest child must sit between the mother and father.

How many different ways can they sit?
(1 point)

A) 24 B) 20 C) 12 D) 6 E) 4

Q2:

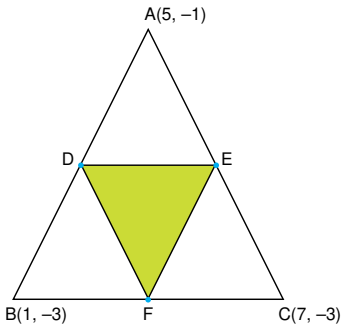


In the figure, ABCD is a square, and the circle with center O is tangent to the sides of the square. It is given that IABI = 12 cm.

What is the shaded area of the region between the square and the circle? (2 points)

A) $144 - 18\pi$ B) $144 - 36\pi$
C) $72 + 18\pi$ D) $72 - 18\pi$
E) $72 - 36\pi$

Q3:

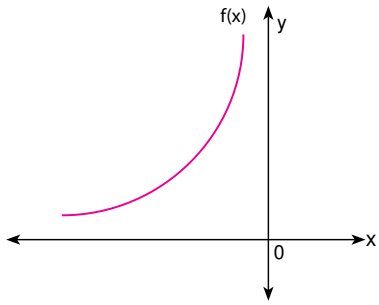


In the triangle ABC shown above, the points D, E, and F are the midpoints of the sides of triangle ABC.

What is the area of triangle DEF? (3 points)

A) $\frac{3}{2}$ B) 2 C) $\frac{5}{2}$ D) 3 E) $\frac{7}{2}$

Q4:



In the figure above, the graph of the function y = f(x) is given.

Based on the graph:

- I. f(x) is increasing.
- II. f(x) is negative.
- III. f(0)=0.

Which of the following statements are true?
(4 points)

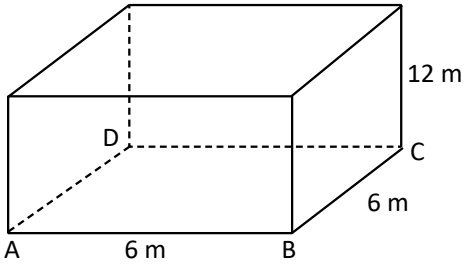
A) Only I B) Only II
C) I and II D) I and III
E) II and III

Q5: $\dfrac{\cos^2 x + \sin^2 x + 1}{1 - \sin^2 x}$

Which of the following is the simplest form of the expression? (5 points)

A) 2 B) 2cotx
C) $\tan^2 x$ D) –2
E) –2tanx

Q6:

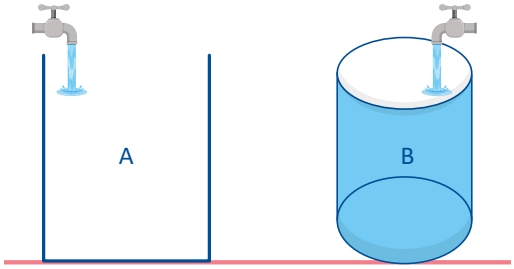


In the figure, a square pyramid is placed inside a rectangular prism, using the ABCD face as the base of the pyramid.

Given this information, what is the volume of the square pyramid? (6 points)

A) 100 cm^3 B) 110 cm^3
C) 115 cm^3 D) 120 cm^3
E) 144 cm^3

Q7:



The volume of tank A is $x^3 - x^2 - 6x$ liters, and the volume of tank B is $3x^2 + 9x$ liters.

A faucet that fills tank A releases water at a rate of x+2 liters per minute and fills the tank completely in m minutes. A different faucet that fills tank B releases water at a rate of x + 3 liters per minute and fills the tank completely in n minutes.

What is the expression for m + n? (7 points)

A) $x^2 - 3x$ B) $x^2 - 1$
C) $x^2 - x$ D) $x^2 - 3$
E) x^2

SOLUTIONS

SOLUTIONS 3-4

ANSWER IS B

SOLUTION:

Q1: We are looking for the largest even number less than 500 with all unique digits.

Start with the largest possible even number under 500, which is 498.

- Digits: 4, 9, and 8 (all different)

The largest even natural number less than 500 whose digits are all different from each other is B) 498.

ANSWER IS C

SOLUTION:

Q3: The total number of attendees was 845. The number of attendees in the morning was 342, and in the afternoon, it was 276. Calculate the total number of attendees in the morning and afternoon:

$342 + 276 = 618$

Subtract this sum from the total number of attendees to find the number of attendees in the evening:

$845 - 618 = 227$

The number of people who attended the concert in the evening is

C) 227.

ANSWER IS D

SOLUTION:

Q2: The known digits are 3 and 5.

The sum of these known digits is $3 + 5 = 8$.

The equation for the total sum is:

$3 + \Delta + \Delta + 5 = 20$

Simplify to: $\Delta + \Delta = 12$

The sum of the digits Δ is D) 12.

ANSWER IS A

SOLUTION:

Q4: The perimeter of one square is 20 cm. The side length of one square can be found by dividing the perimeter by 4:

$$\text{Side length} = \frac{20\text{ cm}}{4} = 5\text{ cm}$$

The shape consists of 5 equal squares arranged in a row. The total length of the shape will be the sum of the sides of the squares along the top or bottom row.

In this shape, we have 12 side lengths in total.

$12 \times 5\text{ cm} = 60\text{ cm}$

So, the answer is 60 cm

ANSWER IS B

SOLUTION:

Q5: First, calculate the time from 21:30 to midnight (00:00):

From 21:30 to 24:00 is 2 hours and 30 minutes.

Next, calculate the time from midnight to 07:15:

This is 7 hours and 15 minutes.

Add these two time periods together:

2 hours 30 minutes+7 hours 15 minutes=9 hours 45 minutes

The total time Alex slept is 9 hours 45 minutes.

ANSWER IS A

SOLUTION:

Q6: A rectangle has two long sides and two short sides.

The long sides are 60 meters each,

so: 60 + 60 = 120 meters

The short sides are 40 meters each,

so: 40 + 40 = 80 meters

Add these together to get the total distance around the garden: 120 + 80 = 200 meters

Calculate how many flowers can be planted:

Emma plants a flower every 4 meters.

To find out how many flowers she can plant, divide the total distance by

$$4: \frac{200 \text{ meters}}{4 \text{ meters per flower}} = 50 \text{ flowers}$$

ANSWER IS C

SOLUTION:

Q7: What does "two-twelfths" mean?

- It means if you divide the whole race into 12 equal parts, Chris swam 2 of those parts.
- To find out how many meters that is, divide 240 meters by 12 (which is 20 meters for each part), then multiply by 2 : 20 meters × 2 = 40 meters

What does "one-third" mean?

- It means if you divide the race into 3 equal parts, Taylor swam 1 of those parts.
- Divide 240 meters by 3 : 240 ÷ 3 = 80 meters

What does "one-fifth" mean?

- It means if you divide the race into 5 equal parts, Morgan swam 1 of those parts.
- Divide 240 meters by 5: 240 ÷ 5 = 48 meters

What does "one-sixth" mean?

- It means if you divide the race into 6 equal parts, Jamie swam 1 of those parts.
- Divide 240 meters by 6: 240 ÷ 6 = 40 meters
- Chris swam 40 meters.
- Taylor swam 80 meters.
- Morgan swam 48 meters.
- Jamie swam 40 meters.

Chris and Jamie swam the same distance. So, the correct answer is C.

SOLUTIONS 5-6

ANSWER IS C

SOLUTION:

Q1: To find the pattern, let's check the difference between consecutive numbers:

- $14 - 8 = 6$
- $20 - 14 = 6$
- $26 - 20 = 6$
- $32 - 26 = 6$

The difference between the numbers is consistently 6. Therefore, to find A, we add 6 to 32:

$32 + 6 = 38$

So, $A = 38$.

The correct answer is 38.

ANSWER IS D

SOLUTION:

Q2: **1. Rightmost column** $(5 - C = 6)$**:**

Since 5 minus something results in 6, this indicates that there was borrowing.

So, we treat 5 as 15 (after borrowing from the tens place), and we have:

$15 - C = 6 \Rightarrow C = 9$

2. Middle column $(B - B = C)$**:**

After borrowing, B in the top row is reduced by 1. So, the subtraction becomes:

$(B - 1) - B = 9$

Therefore, B can be any number. The value of B is not important.

3. Leftmost column ($A - 6 = 2$ after borrowing from A)**:**

Since we borrowed 1 from A, the equation becomes:

$(A - 1) - 6 = 2 \Rightarrow A = 9$

Now we know:

- $A = 9$
- $B = 8$
- $C = 9$

Finally, we calculate $A + C$:

$A + C = 9 + 9 = 18$

So, the correct answer is 18.

ANSWER IS A

SOLUTION:

Q3: A mixed number is converted to an improper fraction using the formula:

Improper fraction = A × denominator + numerator

In this case:

$$\frac{37}{5} = A \times 5 + 2$$

Let's solve for A:

Start with the equation:

$37 = A \times 5 + 2$

Subtract 2 from both sides:

$37 - 2 = A \times 5$

$35 = A \times 5$

Divide both sides by 5:

$$A = \frac{35}{5} = 7$$

Thus, the value of A is 7.

ANSWER IS B

SOLUTION:

Q4: We have a square ABCD, and the angle $(\widehat{ADE})$ is given as 31°. Points E, D, and F are on a straight line. We need to find the angle $(\widehat{CDF})$.

In a square, all four corners are 90°. So, $(\widehat{ADC})$, which is one corner of the square, is 90°.

You know that $(\widehat{ADE})$ is 31°. Since E, D, and F are in a straight line, $(\widehat{EDF})$ forms a straight angle, which means it equals 180°. We can subtract the sum of angles ADE and ADC from the straight angle of 180° to find the remaining angle at $(\widehat{CDF})$:

$(\widehat{ADE}) + (\widehat{ADC}) = 31° + 90° = 121°$

$(\widehat{CDF}) = 180° - 121° = 59°$

ANSWER IS A

SOLUTION:

Q5: We know that after Brian drives 10% of his journey, he still has 45 kilometers left to go. That means the 45 kilometers represents the remaining 90% of the journey.

To find the total distance, we can think of the journey in parts. If 90% of the journey is 45 kilometers, then 10% of the journey must be:

$\frac{45}{9}$ = 5 kilometers

Now, if 10% of the journey is 5 kilometers, we can find the total distance by multiplying 10% by 10 to get 100% (the full journey):

5 × 10 = 50 kilometers

So, the total journey is 50 kilometers.

The correct answer is A.

ANSWER IS A

SOLUTION:

Q6: Since the total length of the line segments is 190 cm and there are 38 line segments, the length of each side of the square is:

$\frac{90}{38}$ = 5 cm

So, each side of a square is 5 cm.

The rectangle consists of:

3 squares tall (height),

5 squares wide (width).

Each square has sides of 5 cm, so:

The total height of the rectangle is: 3 × 5 = 15 cm

The total width of the rectangle is: 5 × 5 = 25 cm.

The area of the rectangle is given by multiplying the width by the height:

Area = height × width = 15 cm × 25 cm = 375 cm^2

The area of the rectangle is 375 cm^2, so the correct answer is A.

ANSWER IS B

SOLUTION:

Q7: Let's denote:

- The price of 1 kilogram of apples as a
- The price of 1 kilogram of bananas as b
- The price of 1 kilogram of tomatoes as t

From the problem, we know:

4a = 3b (The price of 4 kg of apples equals the price of 3 kg of bananas),

4a = 5t (The price of 4 kg of apples equals the price of 5 kg of tomatoes).

Step 1: Find the price of 1 kilogram of bananas and tomatoes

- From 4a = 3b, we can solve for b : $b = \frac{4a}{3}$
- From 4a = 5t, we can solve for t : $t = \frac{4a}{5}$

Step 2: Calculate the total price of 1 kilogram of apples and 1 kilogram of bananas

The total price of 1 kilogram of apples and 1 kilogram of bananas is:

$$a + b = a + \frac{4a}{3} = \frac{3a}{3} + \frac{4a}{3} = \frac{7a}{3}$$

Step 3: Find the ratio of the price of 1 kilogram of tomatoes to the total price of 1 kilogram of bananas and apples

Now, we need to find the ratio $\frac{t}{a+b}$:

$$\frac{t}{a+b} = \frac{\frac{4a}{5}}{\frac{7a}{3}} = \frac{4a}{5} \times \frac{3}{7a} = \frac{12}{35}$$

Thus, the correct answer is B.

SOLUTIONS 7-8

ANSWER IS C

SOLUTION:

Q1: We know the following relationships from the problem:

- B is the sum of the two numbers directly below it: $-2 + 3 = 1$
- C is the sum of the two numbers directly below it: $3 + (-5) = -2$
- A is the sum of B and C, so:

  $A = B + C = 1 + (-2) = -1$

Now, we need to calculate A + B + C:

$A + B + C = -1 + 1 + (-2) = -2$

So the correct answer is C.

ANSWER IS C

SOLUTION:

Q2: Let's evaluate the numbers in each column:

Column I:

- $64^1 = 64$
- $1^{64} = 1$

So, the numbers in column I are not equal.

Column II:

- $(-8)^2 = (-8) \times (-8) = 64$
- $4^3 = 4 \times 4 \times 4 = 64$

So, the numbers in column II are equal.

Column III:

- $(-2)^6 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 64$
- $8^2 = 8 \times 8 = 64$

So, the numbers in column III are equal.

Column IV:

- $(-4)^3 = (-4) \times (-4) \times (-4) = -64$
- $(-2)^6 = 64$

So, the numbers in column IV are not equal.

The columns where the numbers are equal are II and III. The correct answer is C

ANSWER IS C

SOLUTION:

Q3: Let's use algebra to solve this. Let x represent the total capacity of the tank in liters.

Initially, the tank has $\frac{2}{7} \times x$ liters of water.

After adding 45 liters, the tank has $\frac{1}{2} \times x$ liters.

So, the equation is: $\frac{2}{7} \times x + 45 = \frac{1}{2} \times x$

First, eliminate the fractions by multiplying the entire equation by 14 (the least common multiple of 7 and 2):

$$14 \times \left(\frac{2}{7} \times x + 45\right) = 14 \times \frac{1}{2} \times x$$

Simplify: $4x + 630 = 7x$

Subtract 4x from both sides: $630 = 3x$

Divide both sides by 3: $x = 210$ liters

ANSWER IS C

SOLUTION:

Q4: We are given the rule ad−bc, and the number in the center of the circle is 24. Let's substitute the given values into the equation.

$ad - bc = 24$

Substitute the values of a, b, c, and d:

$5(x + 1) - (-3)(2x - 1) = 24$

Simplify both parts:

For $5(x + 1)$:

$5(x + 1) = 5x + 5$

For $-(-3)(2x - 1)$:

$-(-3)(2x - 1) = 3(2x - 1) = 6x - 3$

Now, substitute these back into the equation:

$5x + 5 + 6x - 3 = 24$

Combine like terms:

$11x + 2 = 24$

Subtract 2 from both sides:

$11x = 22$

Now, divide by 11:

$x = \frac{22}{11} = 2$

The value of x is 2, so the correct answer is C.

ANSWER IS C

SOLUTION:

Q5: Since DF // BE and DE // BC, the triangles ADF, ABE, and ABC are similar. This means the corresponding sides of these triangles are proportional.

In triangle ABC, the line segments AF, FE, and EC are given:

- AF = 6 cm
- FE = 12 cm
- EC = x cm

Since the triangles are similar, the proportional relationship holds. Let's now calculate the total length of AC, which is the sum of these segments:

$$AC = AF + FE + EC = 6 + 12 + x$$

The ratio between AF and FE is:

$$\frac{AF}{FE} = \frac{6}{12} = \frac{1}{2}$$

This ratio applies to the remaining segment EC, meaning EC should be twice as long as FE, since the proportional relationship continues along the length of the triangle. Thus:

$$EC = 2 \times (AF + FE)$$

Substituting the known values:

$$EC = 2 \times (6 + 12) = 2 \times 18 = 36\,cm$$

The length of EC, or x, is 36 cm.

ANSWER IS B

SOLUTION:

Q6: The volume of a cylinder is given by the formula: $V = \pi r^2 h$ where:

- r is the radius of the base
- h is the height of the cylinder
- π is a constant.

The first cylinder has a radius of 3 cm and a height of 20 cm. So, its volume is:

$$V_1 = \pi 3^2 \times 20 = \pi \times 9 \times 20 = 180\pi\,cm^3$$

The second cylinder has a base radius of 6 cm, and we need to find its height h. Since the water fills the second cylinder completely, the volume of the second cylinder must be equal to the volume of the first cylinder:

$$V_2 = V1 = 180\pi\ cm^3$$

The volume of the second cylinder is also given by the formula:

$$V_2 = \pi 6^2 \times h = \pi \times 36 \times h = 36\pi$$

Since $V_2 = V_1$, we can set the volumes equal to each other:

$$36\pi h = 180\pi$$

Canceling π from both sides:

$$36h = 180$$

Now, divide both sides by 36:

$$h = \frac{180}{36} = 5\,cm$$

The height of the second cylinder is 5 cm, so the correct answer is B.

ANSWER IS C

SOLUTION:

Q7: We are given two lines:

- $y - x = 2$, or equivalently $y = x + 2$
- $y - x = 4$, or equivalently $y = x + 4$

The lines are parallel and the region between them forms a parallelogram-like shape, but we will approach it by considering two right triangles instead.

X–axis intercepts:

Set $y = 0$ to find the x–intercepts.

- For $y = x + 2$:

  $0 = x + 2 \Rightarrow x = -2$

  So, the x-intercept is (–2,0).

- For $y = x + 4$:

  $0 = x + 4 \Rightarrow x = -4$

  So, the x-intercept is (–4, 0).

Y–axis intercepts:

Set $x = 0$ to find the y–intercepts.

- For $y = x + 2$, when $x = 0$: $y = 2$.

  So, the y–intercept is (0,2).

- For $y = x + 4$, when $x = 0$: $y = 4$.

  So, the y–intercept is (0, 4).

For the line y = x + 2:

- The x–intercept is (–2, 0)
- The y–intercept is (0, 2)

For the line y = x + 4:

- The x–intercept is (–4, 0)
- The y–intercept is (0, 4)

By focusing on the two triangles formed between the lines and the axes, we get:

Triangle 1:

This is the triangle formed by the points (–2,0), (0,2), and the origin (0,0).

- The base of the triangle (along the x–axis) is 2 units (from $x = -2$ to $x = 0$).
- The height of the triangle (along the y–axis) is 2 units (from $y = 0$ to $y = 2$).

The area of Triangle 1 is:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2 \times 2 = 2 \text{ square units}$$

Triangle 2:

This is the triangle formed by the points (–4, 0), (0, 4), and the origin (0, 0).

- The base of the triangle (along the x-axis) is 4 units (from $x = -4$ to $x = 0$).
- The height of the triangle (along the y-axis) is 4 units (from $y = 0$ to $y = 4$).

The area of Triangle 2 is:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 4 = 8 \text{ square units}$$

The shaded region is the difference between the area of Triangle 2 and Triangle 1. So, the total shaded area is:

Shaded Area = 8 – 2 = 6 square units.

The area of the shaded region is 6 square units, so the correct answer is C.

SOLUTIONS 9-10

ANSWER IS C

SOLUTION:

Q1: We need to find the ratio of:

The product of eight 2's

The sum of sixty–four 2's

The product of eight 2's means multiplying 2 by itself eight times:

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 28 = 256$$

The sum of sixty-four 2's means adding 2 together sixty-four times:

$$2 + 2 + 2 + \ldots + 2 = 64 \times 2 = 128$$

The ratio of the product of eight 2's to the sum of sixty-four 2's is:

$$\frac{2^8}{64} \text{ x } 2 = \frac{256}{128}$$

The correct answer is C.

ANSWER IS A

SOLUTION:

Q2: We are given two conditions:

The arithmetic mean of x, y, and z is 9. This can be written as:

$$\frac{x + y + z}{3} = 9$$

4 pt Multiply both sides by 3:

$$x + y + z = 27$$

The arithmetic mean of x and y is 11.

This can be written as:

$$\frac{x + y}{2} = 11$$

Multiply both sides by 2:

$$x + y = 22$$

We have two equations:

$$x + y + z = 27$$

$$x + y = 22$$

Subtract equation 2 from equation 1:

$(x + y + z) - (x + y) = 27 - 22$,

which simplifies to: $z = 5$.

ANSWER IS A

SOLUTION:

Q3: The total area of the large rectangle is given as $(x^2 + 8x + 15)$. This rectangle is divided into four smaller rectangles, and we are specifically interested in the two yellow regions.

The dimensions of the large rectangle are:

- Height = x + 3,
- Width = x + 5, since:

$(x + 3)(x + 5) = x^2 + 8x + 15$

There are two yellow regions in the figure:

- The yellow rectangle at the top-left has dimensions 3 × x, so its area is: 3x
- The yellow rectangle at the bottom-right has dimensions x × 5, so its area is: 5x

The total area of the yellow regions is the sum of the two areas:

Total yellow area = 3x + 5x = 8x. Thus, P(x) = 8x.

Now, substitute x + 4 into the polynomial P(x) = 8x:

$P(x + 4) = 8(x + 4)$

First, expand the terms:

$8(x + 4) = 8x + 32$

The correct option is A.

ANSWER IS A

SOLUTION:

Q4: $\sin 60°: \frac{\sqrt{3}}{2}$

$\cos 45°: \frac{\sqrt{2}}{2}$

$\cos 30°: \frac{\sqrt{3}}{2}$

$\sin 45°: \frac{\sqrt{2}}{2}$

Now, substitute the values of the trigonometric functions into the expression:

$$\frac{\sin 60° + \cos 45°}{\cos 30° + \sin 45°} = \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}}$$

The numerator and the denominator are identical, so the expression simplifies to:

$$\frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}} = 1$$

ANSWER IS D

SOLUTION:

Q5: Cross-multiplying the terms to eliminate the denominators gives:

$(2x + 1)(3x + 1) = (x + 3)(x + 4)$

Now expand both sides:

- Left side:

  $(2x + 1)(3x + 1) = 2x(3x) + 2x(1) + 1(3x) + 1(1)$
  $= 6x^2 + 2x + 3x + 1 = 6x^2 + 5x + 1$

- Right side:

  $(x + 3)(x + 4) = x(x) + x(4) + 3(x) + 3(4)$
  $= x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

Now, we have:

$6x^2 + 5x + 1 = x^2 + 7x + 12$

Subtract $x^2 + 7x + 12$ from both sides:

$6x^2 + 5x + 1 - (x^2 + 7x + 12) = 0$

Simplifying:

$6x^2 - x^2 + 5x - 7x + 1 - 12 = 0$

$5x^2 - 2x - 11 = 0$

In a quadratic equation of the form $ax^2 + bx + c = 0$, the product of the roots is given by:

Product of roots $= \frac{c}{a}$

For the equation $5x^2 - 2x - 11 = 0$

- $a = 5$,
- $c = 11$

Thus, the product of the roots is: $\frac{11}{5}$

ANSWER IS E

SOLUTION:

Q6: We will split the terms into two groups:

Terms with a denominator of 13:

$$\frac{24}{13} + \frac{22}{13} + \frac{20}{13} + \ldots + \frac{2}{13}$$

Terms with a denominator of 12:

$$\frac{23}{12} + \frac{21}{12} + \ldots + \frac{1}{12}$$

The terms with a denominator of 13 form an arithmetic sequence:

$$\frac{24}{13} + \frac{22}{13} + \frac{20}{13} + \ldots + \frac{2}{13}$$

This sequence has 12 terms, with a common difference of -2, and the first term is $\frac{24}{13}$ and the last term is $\frac{2}{13}$.

The sum of these terms is:

$S = \frac{n}{2} \times$ (first term + last term), where $n = 12$, the number of terms. Substituting the values:

$$S = \frac{12}{2} \times \left(\frac{24}{13} + \frac{2}{13}\right) = 6 \times \frac{26}{13} = 6 \times 2 = 12$$

The terms with a denominator of 12 form another arithmetic sequence:

$$\frac{23}{12} + \frac{21}{12} + \ldots + \frac{1}{12}$$

This sequence also has 12 terms, with a common difference of -2, and the first term is 23/12 and the last term is $\frac{1}{12}$.

The sum of these terms is:

$$S = \frac{12}{2} \times \left(\frac{23}{12} + \frac{1}{12}\right) = 6 \times \frac{24}{12} = 6 \times 2 = 12$$

Now, adding the sums of the two groups:

$12 + 12 = 24$.

The correct answer is 24, so the correct option is E.

ANSWER IS C

SOLUTION:

Q7: We are given a rectangular prism with the following dimensions:

- Length = 4 units
- Width = 5 units
- Height = 6 units

The entire surface of this prism is painted. After the block is cut into unit cubes, we need to find how many of these cubes will have no painted faces. These cubes are the ones completely inside the block, not touching the outer surface. The total number of unit cubes is simply the product of the dimensions of the rectangular prism:

Total number of cubes = $4 \times 5 \times 6 = 120$ unit cubes

The cubes that have no paint are the ones that are not touching the outer surface of the rectangular prism. These cubes are inside the block, and to count them, we must remove the outer layers of cubes from each face.

To do this, we subtract 1 unit from each dimension to remove the outer layer:

- The inner length becomes $4 - 2 = 2$ (since we are removing 1 unit from both sides),
- The inner width becomes $5 - 2 = 3$,
- The inner height becomes $6 - 2 = 4$.

Thus, the number of inner cubes with no painted faces is: Inner cubes = $2 \times 3 \times 4 = 24$

The number of unit cubes with no painted faces is 24, so the correct answer is C.

SOLUTIONS 11-12

ANSWER IS C

SOLUTION:

Q1: Since the youngest child must sit between the mother and father, we need to first focus on the three of them occupying three consecutive seats. There are two possible ways to arrange them:

1. Mother – Youngest Child – Father
2. Father – Youngest Child – Mother

Thus, there are 2 ways to arrange the mother, father, and youngest child in those 3 adjacent seats.

Once the mother, father, and youngest child are seated, there are 2 remaining seats for the other two children. These two children can be seated in $2! = 2$ ways.

We have 5 total seats and the group of three (mother, father, and youngest child) can sit in 3 different sets of adjacent seats. These 3 seats can start at positions 1, 2, or 3.

So, the correct total number of seating arrangements is:

Total ways = $3 \times 2 \times 2! = 3 \times 2 \times 2 = 12$

The correct answer is C.

ANSWER IS B

SOLUTION:

Q2: We are given that AB = 12 cm. Since ABCD is a square, all sides are equal, so the side length of the square is 12 cm.

The area A_{square} of the square is:

$A_{square} = s^2 = 12^2 = 144 \text{ cm}^2$

The circle is inscribed inside the square, meaning it touches all four sides of the square. The diameter of the circle is equal to the side length of the square.

The diameter of the circle is 12 cm,

Therefore, the radius r of the circle is: $r = \frac{12}{2} = 6 \text{ cm}$

The area A_{circle} of the circle is given by the formula:

$A_{circle} = \pi r^2 = \pi(6)^2 = 36\pi \text{ cm}^2$

The area of the shaded region is the area of the square minus the area of the circle:

$A_{shaded} = A_{square} - A_{circle} = 144 - 36\pi \text{ cm}^2$

The area of the region between the square and the circle is $144 - 36\pi$ cm^2, so the correct answer is B.

ANSWER IS A

SOLUTION:

Q3: TWe can use the formula for the area of a triangle when the vertices are given as coordinates:

Area of triangle

$= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Substitute the coordinates of A(5, –1), B(1, –3), and C(7,–3):

Area of triangle

$ABC = \frac{1}{2} \ 5(-3 + 3) + 1(-3 + 1) + 7(-1 + 3)$

Simplifying:

$= \frac{1}{2} |5(0) + 1(-2) \ + 7(2)| = 12|0 - 2 + 14|$

$= \frac{1}{2} \times 12 = 6$ square units.

Since D, E, and F are midpoints of the sides of triangle ABC, triangle DEF is the medial triangle of triangle ABC. The area of the medial triangle is always one-fourth of the area of the original triangle.

Thus, the area of triangle DEF is:

Area of triangle DEF $= \frac{1}{4} \times$ Area of triangle ABC $= \frac{1}{4} \times 6 = \frac{3}{2}$

The area of triangle DEF is 3/2 square units, so the correct answer is A.

ANSWER IS A

SOLUTION:

Q4: The graph of y=f(x) is provided, and we need to determine which statements about the function are true based on the graph.

Analyze the statements:

Statement I: f(x) is increasing

From the graph, as x increases, f(x) clearly increases as well. The curve moves upwards as we go from left to right, indicating that the function is indeed increasing.

This statement is true.

Statement II: f(x) is negative

The graph shows that f(x) is entirely above the x-axis for all values of x shown. A function is considered positive when it's above the x-axis. Therefore, the function is positive, not negative.

This statement is false.

Statement III: f(0) = 0

From the graph, it does not appear that f(x) passes through the origin. It seems that f(0) has a positive value, not 0.

This statement is false.

Statement I is true, but both Statements II and III are false.

The correct answer is A.

ANSWER IS A

SOLUTION:

Q5: We know the identity:

$1 - \sin^2 x = \cos^2 x$

So, we can substitute $\cos^2 x$ for $1 - \sin^2 x$ in the denominator of the expression.

Thus, the expression becomes:

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$

The numerator contains $\cos^2 x - \sin^2 x + 1$. We can rewrite the expression$\cos^2 x - \sin^2 x$ as $\cos(2x)$, using the double angle identity for cosine:

$\cos^2 x - \sin^2 x = \cos(2x)$.

Thus, the numerator becomes $\cos(2x) + 1$.

Now the expression becomes:

$$\frac{\cos(2x) + 1}{\cos^2 x}$$

We need to break this into two terms:

$$\frac{\cos(2x)}{\cos^2 x} + \frac{1}{\cos^2 x}$$

The second term is simply:

$$\frac{1}{\cos^2 x} = \sec^2 x$$

For the first term, recall that:

$\cos(2x) = 2\cos^2 x - 1$

So,

$$\frac{\cos(2x) + 1}{\cos^2 x} = 2 - \frac{1}{\cos^2 x}$$

Therefore, the entire expression simplifies to:

$2 - \sec^2 x + \sec^2 x = 2$

The simplified expression is 2, so the correct answer is A.

ANSWER IS E

SOLUTION:

Q6: The formula for the volume V of a pyramid is:

$$V = \frac{1}{3} \times \text{Base Area} \times \text{Height}$$

Since the base of the pyramid is a square with side lengths of 6 meters, the area of the base is:

Base Area = $6 \times 6 = 36$ square meters

The height of the pyramid is the same as the height of the rectangular prism, which is 12 meters.

Now, using the formula for the volume of a pyramid:

$$V = \frac{1}{3} \times 36 \times 12 = \frac{1}{3} \times 432 = 144 \text{ cubic meters}$$

The volume of the square pyramid is 144 cubic meters, so the correct answer is E.

ANSWER IS E

SOLUTION:

Q7: The volume of tank A is $x^3 - x^2 - 6x$ liters, and the faucet releases $x + 2$ liters per minute. The time m to fill tank A is given by:

$$m = \frac{\text{Volume of tank A}}{\text{Flow rate for tank A}} = \frac{x^3 - x^2 - 6x}{x + 2}$$

We can simplify this by performing polynomial division.

Polynomial Division: $\frac{x^3 - x^2 - 6x}{x + 2}$

Step 1: Divide the first term of the numerator by the first term of the denominator: $\frac{x^3}{x^2}$

Now multiply x^2 by $x + 2$:

$x^2(x + 2) = x^3 + 2x^2$

Subtract $x^3 + 2x^2$ from $x^3 - x^2 - 6x$:

$(x^3 - x^2 - 6x) - (x^3 + 2x^2) = -3x^2 - 6x$

Step 2: Divide the first term of the new numerator by the first term of the denominator:

$\frac{-3x^2}{x} = -3x$

Now multiply $-3x$ by $x + 2$: $-3x(x + 2) = -3x^2 - 6x$

Subtract $-3x^2 - 6x$ from $-3x^2 - 6x$:

$(-3x^2 - 6x) - (-3x^2 - 6x) = 0$

Thus, the division is exact, and: $m = {}^2 - 3x$

The volume of tank B is $3x^2 + 9x$ liters, and the faucet releases $x + 3$ liters per minute. The time n to fill tank B is:

$$n = \frac{\text{Volume of tank B}}{\text{Flow rate for tank B}} = \frac{3x^2 + 9x}{x + 3}$$

We can simplify this by factoring the numerator:

$$n = \frac{3x(x + 3)}{x + 3}$$

Cancel the common factor $x + 3$: $n = 3x$

Now, sum the values of m and n:

$m + n = (x^2 - 3x) + 3x = x^2$

The expression for $m + n$ is x^2, so the correct answer is E.